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# A Different Approach in Studying Mean Cordial Labeling of Joins of Bi Star graph

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***Abstract:*** **A graph is defined as set of all vertices and edges. A mapping f from $V(G)$ to $\{0,1,2\}$ is called mean cordial labelling if for each edge $uv$ assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ such that $|v_f(i)-v_f(j)| \le 1$ and $|e_f(i)-e_f(j)| \le 1$ where $i, j \in \{0,1,2\}$. If the graph is mean cordial then it is called mean cordial graph. In this paper we intend to study on the joins of Bi Star graph and prove that they are mean cordial graph. Further we study some properties of joins of Bi Star graph.**

***Keywords: Bi Star graph*, Mean cordial labelling, Mean Cordial graph, Joins of Bi Star graph.**

## I. INTRODUCTION

A nonempty set with set of all vertices and edges together is called a graph and we consider in our discussion graph which are undirected and for all the preliminary concepts we refer to Hand book of graph theory[1]. An extensive study of labelling was given by GallianJ.A[2]. In dynamic survey. Mean Cordial labelling of graph is one such labelling techniques introduced by Ponraj.R, Sivakumar.M and Sundaram.M[3,4]. Some more graphs are studied for Mean Cordial labelling [5]We wish to study in this paper on joins of Bi Star graph. A Bi Star graph is joined with another Bi Star graph by an edge and is called 1-Join of Bi Star graph. Similarly we extend our study on M-Joins of Bi Star graph. We also wish to study on some properties connecting the joins of Bi Star graph in this paper.

## II. PRELIMINARIES

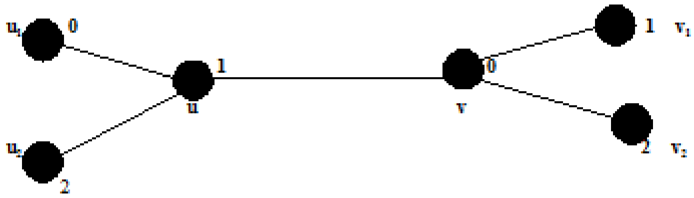
**Definition .2.1**: A mapping f from $V(G)$ to $\{0,1,2\}$ is called a mean cordial labelling if for each edge $uv$ assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ such that $|v_f(i)-v_f(j)| \le 1$ and $|e_f(i)-e_f(j)| \le 1$ where $i, j \in \{0,1,2\}$. If the graph is mean cordial then it is called mean cordial graph

**Definition .2.2**: Bi star $B_{n,n}$ graph is obtained from two disjoint copies of $K_{1,n}$ by joining the centre vertices through an edge.

## III. MAIN RESULTS

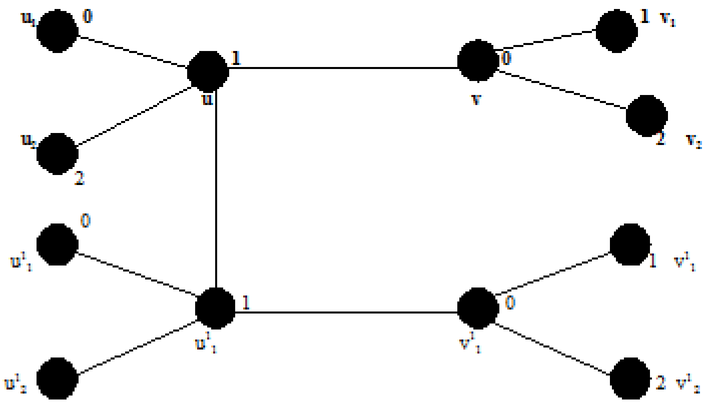
We know a Bi Star Graph $B(2,2)$ given below consists of 4 vertices and 5 edges for which the vertices are labelled with $\{0,1,2\}$ as follows

**Figure.1 :Labeling of Bi Star graph** $B(2,2)$




From the labels assigned for the Bi-Star Graph $B(2,2)$ we understand that two vertices are labelled with 0's, two vertices are labelled with 1's and two vertices are labelled with 2's and the induced edges thus labelled with three 0's and two 1's turns to be a mean cordial labelling graph. Now let us join a Bi Star Graph $B(2,2)$ with another Bi Star Graph $B(2,2)$ by an edge. The graph in such a way obtained is called a 1- Join of Bi Star Graph $B(2,2)$. Similarly we construct 2- Join Bi Star graph $B(2,2)$ by attaching edge between 1-Join Bi Star graph $B(2,2)$ with a Bi Star graph $B(2,2)$. Continuing in this fashion we obtain M-joins of Bi Star graph $B(2,2)$.

**Figure .2 :Labeling of 1- Join of Bi Star graph** $B(2,2)$



**Note**: Here we consider the Bi Star graph joined with Bi Star graph of the same order. i.e if we consider a Bi Star graph $B(2,2)$ then we attach with another Bi Star graph $B(2,2)$.

Now let us prove that 1- Join of Bi Star graph $B(2,2)$ is mean cordial graph in the following theorem.

**Theorem.3.1**: 1- Join of Bi Star graph $B(2,2)$ is mean cordial graph

**Proof**: Let G= 1- Join of Bi Star graph $B(2,2)$. We know from the construction of 1- join of Bi Star graph $B(2,2)$ that the vertex set is given by $V(G)=\{u,v,u_1,u_2,v_1,v_2\}\cup\{u^1,v^1,u^1{}_1,u^1{}_2,v^1{}_1,v^1{}_2\}$

and the edge set is given by $E(G)=\{uv\}\cup\{uu_1,uu_2,vv_1,vv_2\}\cup\{u^1u^1{}_1,v^1v^1{}_1,u^1u^1{}_2,v^1v^1{}_2\}$

Now let us label the vertices of the graph G= 1- Join of Bi Star graph $B(2,2)$

$f(u)=f(u^1)=1$

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$$f(v) = f(v^1) = 0$$

$$f(u_1) = f(u^1{}_1) = 0$$

$$f(u_2) = f(u^1{}_2) = 2$$

$$f(v_1) = f(v^1{}_1) = 1$$

$$f(v_2) = f(v^1{}_2) = 2$$

Then the induced edge labelling of the graph is such that

$$f^*(uv) = f^*(u^1v^1) = 0$$

$$f^*(uu_1) = f^*(u^1u^1{}_1) = 0$$

$$f^*(uu_2) = f^*(u^1u^1{}_2) = 1$$

$$f^*(vv_1) = f^*(v^1v^1{}_1) = 0$$

$$f^*(vv_2) = f^*(v^1v^1{}_2) = 1$$

$$f^*(uu^1) = 1$$

From the above induced edge labelling we find that the number of edges labelled with 0's is 6 and the number of edges labelled with 1's is 5 and hence $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$ where $i, j \in \{0,1,2\}$ which implies that the graph G is mean cordial graph. Hence the proof of the theorem.

**Observation.1**: From the above proof of the theorem.3.1 we find that the labels assigned for the vertices of first Bi Star graph $B(2,2)$ to that of second Bi Star graph $B(2,2)$ to form 1-Join of Bi Star graph $B(2,2)$ is copy of one to another. i.e the vertices have the same labelling.

Hence it follows that by joining as many number of Bi Star graph $B(2,2)$ to form joins of Bi Star graph $B(2,2)$ with the same labelling for the vertices for each of the Bi Star graph $B(2,2)$ we can prove that M-Joins of Bi Star graph $B(2,2)$ is mean cordial graph.

The following table illustrates the number of vertices and edges labelled with 0's, 1's and 2's of Joins of Bi Star graph $B(2,2)$

| **No. of Joins of Bi Star graph $B(2,2)$** | | **Number of 0's** | **Number of 1's** | **Number of 2's** |
|---|---|---|---|---|
| Basic Bi Star graph $B(2,2)$ | No of Vertices | 2 | 2 | 2 |
| | No. of Edges | 3 | 2 | ---- |
| 1-Join of Bi Star graph $B(2,2)$ | No of Vertices | 4 | 4 | 4 |
| | No. of Edges | 6 | 5 | ---- |
| 2-Join of Bi Star | No of Vertices | 6 | 6 | 6 |

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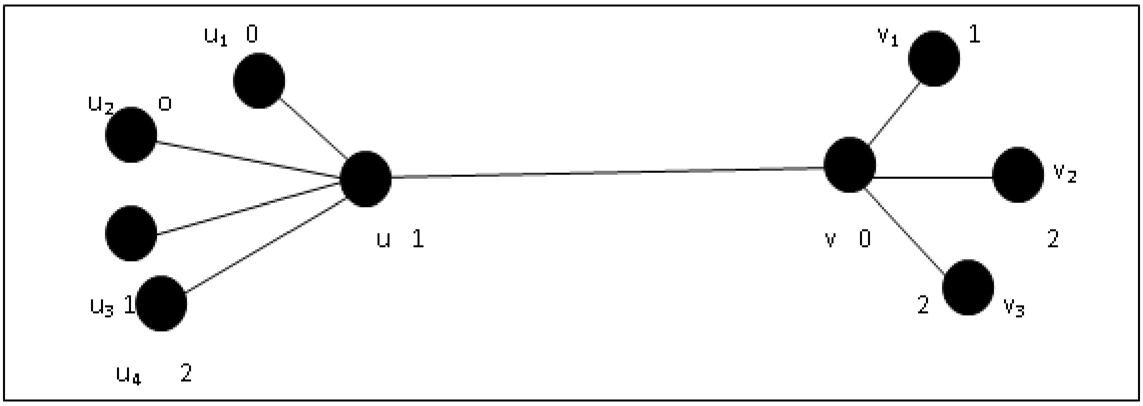
| | | | | |
|---|---|---|---|---|
| graph $B(2,2)$ | No. of Edges | 8 | 8 | ----- |
| 3- Join of Bi Star graph $B(2,2)$ | No. of Vertices | 8 | 8 | 8 |
| | No. of Edges | 12 | 11 | ---- |

**Note.1**: From the above table we find that the number of vertices labelled with 0,1,2 are all equal for the basic Bi Star graph $B(2,2)$, 1- Join of Bi Star graph $B(2,2)$ and so on.

**Note.2:** By the labelling techniques followed in theorem.3.1 it is also found that the number of induced edges labelled with 0's is one greater than the number of edges labelled with 1's for the basic Bi Star graph $B(2,2)$, 1- Join of Bi Star graph $B(2,2)$ and so on.

Now in the construction process of Joins of Bi Star graph we try to fix the number of vertices labelled with 0 as 3 , the number of vertices labelled with 1 as 3 and the number of vertices labelled with 2 as 3. In the way of constructing such a Bi Star graph G we find that the total number of vertices to be 9 and the total number of edges connecting them to form a Bi Star graph is 8. So by virtue we have the Bi Star graph to be in either of the form as $B(4,3)$ or $B(3,4)$ which consists of 9 vertices and 8 edges as illustrated below.

**Figure.3**: Labeling of Bi Star graph $B(4,3)$




Now to label the vertices of the Bi Star graph $B(4,3)$ we follow the following techniques.

As the number of vertices labelled with 0's is 3 we attach to vertex u labelled with 1 of the Bi Star graph $B(4,3)$ with two vertices $u_1, u_2$ labelled each with 0 and attached by edges joining $uu_1, uu_2$ and the remaining vertex $u_3$ is labelled with 1 and vertex $u_4$ is labelled with 2. The vertex v labelled with 0 of the Bi Star graph $B(4,3)$ connected with the vertex $v_1$ labelled with 1 by an edge $vv_1$ and the two vertices $v_2, v_3$ labelled each with 2 by edges $vv_2, vv_3$. The following is the resulting Bi Star graph $B(4,3)$.

Now following the labelling given above for the vertices the induced edge labelling of the Bi Star graph $B(4,3)$ is as follows

$$f^*(uv) = 0$$

$$f^*(uu_1) = 0$$

$f^*(uu_2) = 0$

$f^*(uu_3) = 1$

$f^*(uu_4) = 1$

$f^*(vv_1) = 0$

$f^*(vv_2) = 1$

$f^*(vv_3) = 1$

Hence the number of induced edges of the Bi Star graph $B(4,3)$ has the following properties

Namely the number of edges labelled with 0's is 4 and number of edges labelled with 1's is 4 satisfying the condition $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ where $i, j \in \{0,1,2\}$ and hence the Bi Star graph $B(4,3)$ is mean cordial graph.

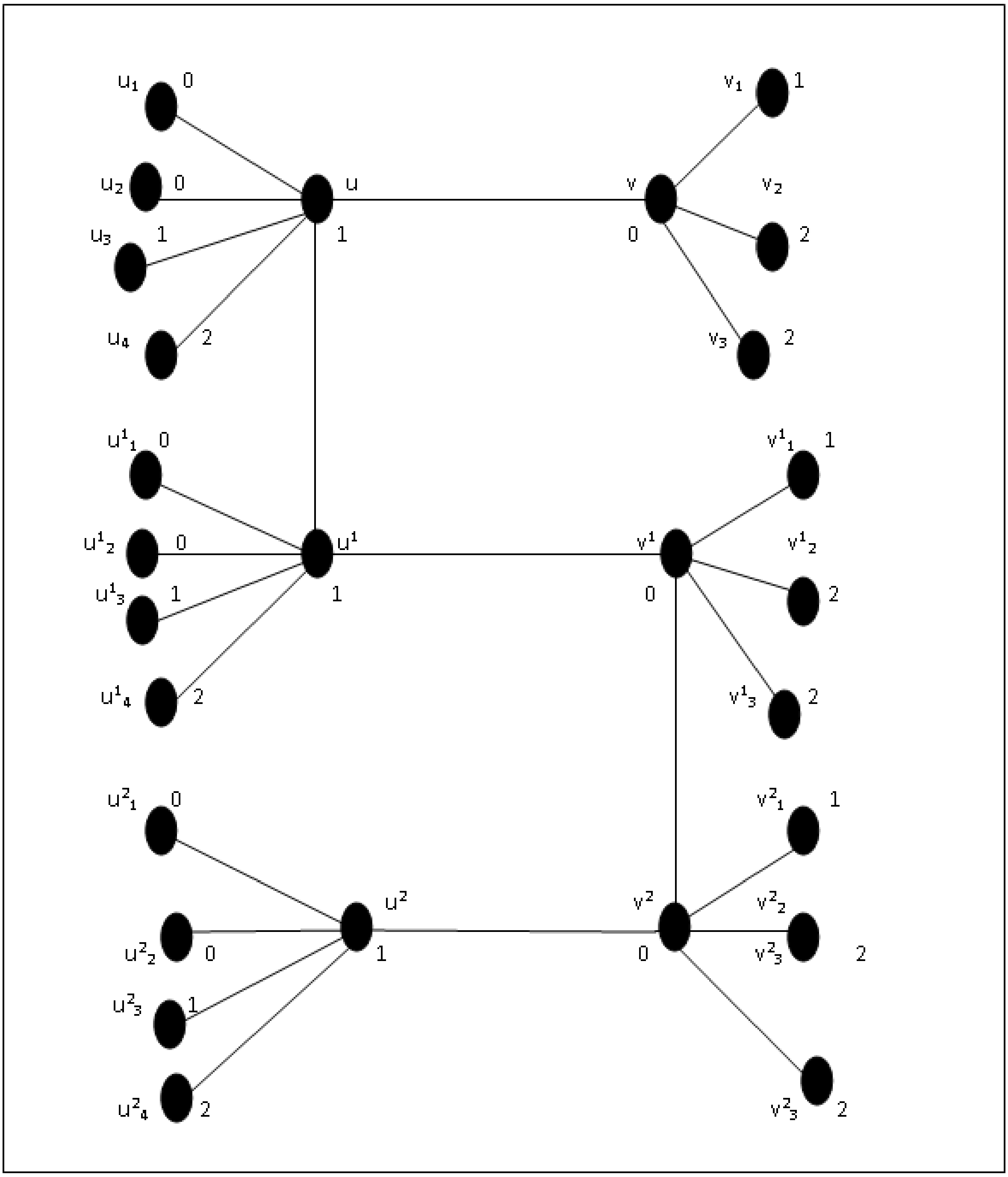
Now let us construct joins of Bi Star graph $B(4,3)$ as constructed for the Bi Star graph $B(2,2)$. In a similar way as like joins of Bi Star graph $B(2,2)$ we find that for any M-Joins of Bi Star graph $B(4,3)$ is mean cordial graph. The number of vertices labelled with 0,1,2 and the number of edges labelled with 0,1 is given in the following table

| **No. of Joins of Bi Star graph $B(4,3)$** | | **Number of 0's** | **Number of 1's** | **Number of 2's** |
|---|---|---|---|---|
| Bi Star Graph $B(4,3)$ | No. of Vertices | 3 | 3 | 3 |
| | No. of Edges | 4 | 4 | ---- |
| 1-Join of Bi Star graph $B(4,3)$ | No of Vertices | 6 | 6 | 6 |
| | No. of Edges | 8 | 8+1=9 | ---- |
| 2-Join of Bi Star graph $B(4,3)$ | No of Vertices | 9 | 9 | 9 |
| | No. of Edges | 12+1=13 | 13 | ----- |
| 3- Join of Bi Star graph $B(4,3)$ | No. of Vertices | 12 | 12 | 12 |
| | No. of Edges | 17 | 17+1=18 | ---- |

The below is the diagram that illustrates the joins of Bi Star graph $B(4,3)$ which is constructed by joining Bi Star graph $B(4,3)$ with another Bi Star graph $B(4,3)$ by an edge. We follow the labelling techniques as specified for the basic Bi Star graph by taking into account the number of 0's,1's and 2's required to label the vertices .

**Note**: To create a Join between the Bi Star graph $B(4,3)$ with another we join the vertex u of first Bi Star graph $B(4,3)$ with vertex $u^1$ of second Bi Star graph $B(4,3)$ and for the 2-join of Bi Star graph $B(4,3)$ we join the vertex $v^1$ with the vertex $v^2$ and so on in order to balance the labels assigned to the vertices so as to prove the Bi Star graph as Cordial labelling graph

**Figure.4**: Labeling of 2-Join of Bi Star Graph $B(4,3)$




Now to construct Joins of Bi Star graph by fixing the number of vertices labelled with 0 as 4 , the number of vertices labelled with 1 as 4 and the number of vertices labelled with 2 as 4. In the way of constructing such a Bi Star graph G we find that the total number of vertices to be 12 and the total number of edges connecting them to form a Bi Star graph is 11. So by virtue we have the Bi Star graph to be in either of the form as $B(5,5)$ which consists of 12 vertices and 11 edges.

Now by labelling in a similar process as in Bi Star graph of the form $B(4,3)$ suitably adding labels for the vertices as 0,1,2 in accordance with the number required we find the following table illustrating the number of vertices labelled with 0,1,2 and number of edges labelled with 0,1 for Bi Star graph $B(5,5)$ and for joins of Bi Star graph $B(5,5)$.

| **No. of Joins of Bi Star graph $B(5,5)$** | | **Number of 0's** | **Number of 1's** | **Number of 2's** |
|---|---|---|---|---|
| Bi Star Graph $B(5,5)$ | No. of Vertices | 4 | 4 | 4 |
| | No. of Edges | 6 | 5 | ---- |
| 1-Join of Bi Star graph $B(5,5)$ | No of Vertices | 8 | 8 | 8 |
| | No. of Edges | 12 | 10+1=11 | ---- |
| 2-Join of Bi Star graph $B(5,5)$ | No of Vertices | 12 | 12 | 12 |
| | No. of Edges | 18 | 16+1=17 | ----- |
| 3- Join of Bi Star graph $B(5,5)$ | No. of Vertices | 16 | 16 | 16 |
| | No. of Edges | 24 | 22+1=23 | ---- |

Continuing this way we can obtain the different types of Bi Star graph by fixing the number of 0's,1's and 2's and hence obtain the induced edge labelling for the corresponding Bi Star graph to be called mean cordial graph.

Hence the following theorem can be claimed

**Theorem.3.2** : For a Bi Star graph by fixing the number of vertices labelled with 0,1,2 each as n , the number of edges of the resulting graph is n-1.

**Proof**: Let us consider the Bi Star graph and fix the number of vertices labelled with 0,1,2 each as n. Now to claim that the number of edges of the resulting graph is n-1. Let us prove the result by using Principle of Mathematical induction on n the number of vertices labelled with 0,1,2. Suppose n=2 then according to the hypothesis we have 2 vertices labelled with 0, 2 vertices labelled with 1, 2 vertices labelled with 2 and hence the total number of vertices of the Bi Star graph is 6. Now by understanding the nature of Bi Star graph we find that the possible Bi Star graph with number of vertices 6 is $B(2,2)$ which consists of 5 edges and hence the theorem holds good for n=2. Now let us assume that the theorem is true for n=k i.e we assume that the number of edges of the resulting graph is k-1. Now to prove for n=k+2 ( we are proving for n=k+2 since the basic Bi Star graph is $B(2,2)$ ) i.e to prove for that the resulting graph has number of edges as k+1. Considering the basic Bi Star graph $B(2,2)$ which consists of 5 edges and we have assumed that the theorem holds good for n=k. Hence from the basic Bi Star graph and the resulting Bi Star graph for n=k the theorem is true for n=k+2. Hence the proof.

**Theorem.3.3**: For a Bi Star graph $B(a,b)$ by fixing the number of vertices labelled with 0,1,2 each as n with the condition $a+b=n-1$ the required Bi Star graph is either of the form $B(a,(n-1)-a)$ or $B((n-1)-a,a)$.

**Proof:** Consider the Bi Star graph $B(a,b)$ by fixing the number of vertices labelled with 0,1,2 each as n with the condition a+b=n-1. It is understood from our labelling techniques for the vertices that the possible nature of Bi Star graph thus obtained is either of the form $B(a,(n-1)-a)$ or $B((n-1)-a,a)$. Hence the proof.

**Result.1**: For Bi Star graph $B(a,(n-1)-a)$ is mean cordial graph with the condition $a+b=n-1$ and number of vertices labelled with 0,1,2 each as n.

**Result.2**: For M-Joins of Bi Star graph $B(a,(n-1)-a)$ is mean cordial graph with the condition $a+b=n-1$ and number of vertices labelled with 0,1,2 each as n.

## IV. CONCLUSION

We in this paper have identified Bi Star graph and have proved that joins of Bi Star graph is mean cordial graph and have approached in a different way to obtain Bi Star graph which can be labelled to be mean cordial labelling. We intend to study on some more graphs which can also be claimed to be mean cordial graph

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